WP5: Fundamentals of radar interferometry I: One baseline Stage 2 report

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1 Introduction

This document lays out the principle groundwork for radar interferometry with one baseline, that is, two passive antennas and an external transmitting antenna, with the one baseline between the passive antennas. The receive antennas are assumed to be close to the transmitting antennas. The case of remote receiver interferometry is not discussed in this report.

Future reports in the series will develop the technique with several baselines and the consequent application to imaging, and the problems and procedures associated with phase accuracy and phase calibration necessary for imaging.

2 Terminology

Some terminology follows:

1) element — every (crossed dipole) antenna of the array.

2) module — for interferometry, as is explained in the next section, it is necessary to obtain beam-formed data (see below) from multiple groups of elements within the antenna array. For example, in the Jicamarca radar, groups of 12×12 crossed dipoles are combined electrically to form vertical beams from this sub-array. This grouping is called a "module". For EISCAT_3D a less rigid organization should be used, but we will still use the term "module" for a subset of the full array with the capability to produce beam-formed data. To distinguish data produced by a module from that produced by the full array, the former will be called "sub-beam-formed data".

3) gain — In this report, since amplitude-domain data is considered, the antenna gain referred to is the *amplitude* gain, not the power gain usually used in the literature. The power gain is the square of the amplitude gain. Beam width is also modified accordingly.

In the report by WP8, the following data types (and some others) were defined:

a) Sample-level data — these consist of the complex-amplitude data coming from each element of the receiver array. These data are produced by the first-stage digitisation of the received signal. They are time-sequenced and are without integration. It is anticipated that these data will not be stored. The handling of sample-level data falls within the remit of Work Package 4; data will only enter into the data system developed under Work Package 8 after beam-forming.

b) Beam-formed data — these consist of the weighted sum of a number of streams of sample level data, each of which is multiplied by a different complex-valued weighting coefficient in order to form a "beam". Like sample-level data, these data are time-sequenced, are without integration, remain uncorrelated, and still have a time resolution at the sub-IPP level. At the receive-only sites a number of such beam-formed data streams will be generated concurrently. Discussions of the precise interface and format specifications under which beam-formed data will enter the WP8 data system are underway elsewhere within the design study.

The data rate will depend on several aspects of the data acquisition strategy. These include whether the data are split into ion-line and plasma-line streams (with different bandwidths). Thus the design of the data handling system at this level will depend on the specifications which come forth from WP4 and from WP9, which will be handling the signal processing tasks including beam-forming, data decimation, filtering and correlation. However, the data rate from a complete beam, will not exceed 40 Msamples/s (16-bit samples), thus resulting in rates of 288 Gbytes/hr/beam. This assumes that allowance has been made for the fact that the radar will not be transmitting with 100% duty cycle.

c) Correlated data — these data constitute the outputs from the signal processing tasks carried out by WP9, comprising the auto-correlated power domain data from each beam. A correlated data set will generally be appreciably larger than a beam-formed data set, because correlation will involve generating the whole lag profile matrix. However, this inflation in size may be mitigated by post-integrating in time (typically in the order of 1 second). There is no requirement that correlated data from all experiments should have the same time resolution — the time resolution will depend on the properties of the experiment (as at present).

For interferometry, the signal processing tasks carried out by WP9 must be augmented with the capability to produce auto-correlated power domain data from *each beam* of *each module*, as well as cross-correlated power domain data from *each beam* for *each pair of modules*.

With N modules, this processing comprises N^2 as much computations as required for ordinary correlated data alone. (N times as many auto-correlated streams of equal computational burden, and $N(N-1)/2$ times as many cross-correlated streams of roughly twice the burden.)

The interferometric data can probably be discarded unused under most circumstances. Ordinary incoherent scattering has not shown tendencies to produce observable coherence in two-antenna interferometric observations made at the ESR, and this is expected to be the case in most of the observation time.

On the other hand, when significant coherences *are* observed, it will be necessary to store all of these auto-correlated and cross-correlated streams, at 0.1 s time resolution or better, *or* all of the sub-beam-formed data that were used to produce them. In either case, such occurrences will probably result in the requirements for this storage to exceed that of the remaining system by a large factor.

For correlated data, the data expansion factor is N^2 times the time resolution expansion (e.g. if ordinary correlated data is taken at 1 s resolution and interferometry data at 0.1 s, then the expansion is by a factor of $10N^2$. For beam-formed data, since it is not integrated, the expansion factor is simply N.

The procedure outlined above implies a near-real-time monitoring of the interferometric data products to decide whether to discard or save these products (or the sub-beam-formed data).

3 Interferometry for horizontal resolution

With phase-coherent sub-beam-formed data from two modules, there is the opportunity of detecting small-scale structures using an interferometric technique. The technique is in principle the same as that used by (Farley et al., 1981) to infer velocities of travelling eddies in the equatorial electrojet.

When the width of the scatterer is not negligible compared to the antenna beam width, or the receiving antennas have different beam patterns, a more careful treatment of the effect of antenna beam patterns, leading to a more complicated expression, is necessary. The theory was extended to take into account antenna gain patterns in La Hoz et al. (2002), for the case where the transmit antenna was used as one of the receive antennas. This was developed more generally by Grydeland (2003), and published by Grydeland et al. (2004).

To demonstrate the interferometric technique in terms useful for the EISCAT_3D project, we will develop the theory for a two-element radar interferometer following that of Grydeland et al. (2004), but modified to a situation where the transmit antenna is not one of the receive antennas. This approach allows a smoother transition to the more complex case of imaging with a large number of receive modules.

3.1 The received signals

Consider a radar system with two (in general) unequal receiving antenna modules separated by a distance greater than the wavelength of the radar, but much smaller than the distance to a scattering volume to be investigated using this system. The antenna used to transmit (probably the full array) is indicated by the subscript tx, while the receive-only modules are indicated by the subscripts rx, i and rx, j. Define a coordinate system with its centre in the transmitting antenna, pointing in the z direction, and express all linear spatial coordinates in units of wavelength. The discussion is limited to the far field of the antenna modules, whereby integrations over apertures are performed by multiplying with antenna amplitude gain patterns, denoted by $G_{\text{tx}}(x/z, y/z)$ and $G_{\text{rx},i}(x/z, y/z)$, respectively, and phase is computed from the centre point of the module.

The spatial part of the electric field incident on the scattering volume is given by

$$E_{\text{inc}} = \frac{E_0 R_0}{R} G_{\text{tx}}(x/z, y/z) \exp[-2\pi i R] \tag{1}$$

where $R = \sqrt{x^2 + y^2 + z^2}$ is the distance from the transmitting antenna and E_0 is the field at an arbitrary distance R_0. The source term for the scattering is obtained by multiplying this field with a term which is proportional to the electron density fluctuation, which is denoted by $C \cdot n(x, y, z)$. Using the unmodified incident field in this expression implies the assumption of weak scattering, also known as the Born approximation. The constant C describes the scattering strength of the medium and includes among others the electron scattering cross section and the factor $E_0 R_0$ from equation (1).

The spatial part of the scattered field in the aperture plane $z = 0$ is given by

$$E_{\text{sc}}(x, y) = C \iiint \frac{1}{R'} E_{\text{inc}}(x', y', z')\, n(x', y', z')\, g(z' - Z_0) e^{-2\pi i |\boldsymbol{R'} - \boldsymbol{r}|} dx'\, dy'\, dz' \tag{2}$$

where $g(z)$ is a spatial weighting function centered at 0 due to the modulation pattern and impulse response of the radar receiver, $\boldsymbol{r} = (x, y)$ is the position in the aperture plane for which the field is computed, and $|\boldsymbol{R}' - \boldsymbol{r}| = \sqrt{(x' - x)^2 + (y' - y)^2 + z'^2}$ is the distance from the scattering volume element to the point in the aperture plane.

Next, observe that since $g(z)$ is local, and $Z_0 \ggg 1, x, y$, then $1/z$, x/z and y/z are approximately constant over the volume being considered, except when used for phase purposes. We can also make the approximations

$$R = z\sqrt{1 + (x^2 + y^2)/z^2} \approx z(1 + (x^2 + y^2)/2z^2) = z + (x^2 + y^2)/2z \quad (3)$$

and

$$|\boldsymbol{R}' - \boldsymbol{r}| \approx z + (x'^2 + y'^2 + x^2 + y^2 - 2xx' - 2yy')/2z. \quad (4)$$

Insert (1) in (2), and define

$$n_{\boldsymbol{k}}(x, y; Z_0) = \int n(x, y, z) g(z - Z_0) e^{-4\pi i z} dz, \quad (5)$$

which expresses the selection on $\boldsymbol{k}$ which occurs for backscattering, that is, scattering in the direction of the transmitter. We now obtain

$$E_{\rm sc}(x, y) = \frac{C}{Z_0^2} \iint dx'\, dy'\, G_{\rm tx}(x'/Z_0, y'/Z_0) n_{\boldsymbol{k}}(x', y'; Z_0)$$
$$\exp\left[-2\pi i \frac{x'^2 + y'^2}{Z_0}\right] \exp\left[2\pi i \frac{xx' + yy'}{Z_0}\right] \exp\left[-2\pi i \frac{x^2 + y^2}{2Z_0}\right] (6)$$

The received signals in an arbitrary antenna on the ground can now be found by integrating this field over the antenna aperture, or equivalently, by multiplying the expression under the integrals in (6) with gain pattern of the receiving antenna.

Different elementary volumes contribute to the integral with different phase, so the integration over scattering volume is still necessary.

The received signal in the transmitting antenna ($\boldsymbol{r} = \boldsymbol{0}$) is given by

$$f_{\rm tx} = \frac{C}{Z_0^2} \iint n_{\boldsymbol{k}}(x', y'; Z_0) G_{\rm tx}^2 e^{-2\pi i (x'^2 + y'^2)/Z_0} dx'\, dy' \quad (7)$$

In receive-only antenna module i, with gain pattern $G_{{\rm rx},i}$ and displaced a distance $\boldsymbol{r} = (A_i, B_i)$, the signal is

$$f_{{\rm rx},i} = \frac{C}{Z_0^2} \iint dx'\, dy' n_{\boldsymbol{k}}(x', y'; Z_0) G_{\rm tx} G_{{\rm rx},i}$$
$$\exp\left\{-2\pi i \frac{x'^2 + y'^2 - Ax' - By' + (A^2 + B^2)/2}{Z_0}\right\}. \quad (8)$$

3.2 Cross-correlations and autocorrelations

From expressions (7) and (8), form the cross-product and ensemble average, which gives an expression for the complex spatial cross-correlation of the scattering received in two receive modules. To simplify the notation, define

$$\rho_{ij} = \left\langle f_{\mathrm{rx},i} f^*_{\mathrm{rx},j} \right\rangle$$
$$A_{ij} = A_i - A_j, \quad B_{ij} = B_i - B_j, \quad \boldsymbol{D}_{ij} = (A_{ij}, B_{ij})$$

With these definitions, we obtain

$$\begin{aligned}\rho_{ij} = \frac{C^2}{Z_0^4} \iiiint & dx\, dy\, dx'\, dy' \, \langle n_{\boldsymbol{k}}(x, y; Z_0) n^*_{\boldsymbol{k}}(x', y'; Z_0)\rangle \\ & G_{\mathrm{tx}}(x/Z_0, y/Z_0) G_{\mathrm{tx}}(x'/Z_0, y'/Z_0) G_{\mathrm{rx},i}(x/Z_0, y/Z_0) G_{\mathrm{rx},j}(x'/Z_0, y'/Z_0) \\ & \cdot \exp\{-2\pi i(x^2 - x'^2 + y^2 - y'^2 - A_{ij}x' - B_{ij}y')/Z_0\} \\ & \cdot \exp\{2\pi i(A_i^2 + B_i^2 - A_j^2 - B_j^2)/2Z_0\} \end{aligned} \tag{9}$$

The constant phase factor $\exp(i\pi(A_i^2 + B_i^2 - A_j^2 - B_j^2)/Z_0)$ is due to the geometry of the transmitting and receiving antennas. While it is in principle possible to determine it from geometry alone, the actually measured phase offset depends on many aspects of the data acquistion system, several of which will vary slowly with time. It is therefore necessary to measure this phase difference using known signals or scattering targets, a process known as *phase calibration*.

For the purpose of the discussion at hand, it is assumed known, and not regarded in the following. A separate report will deal with the requirements of relative phase accuracy and phase calibration procedures.

Next, assume spatial homogeneity; that the spatial correlations depend only on distance within the scattering volume, not on absolute position. This assumption needs to hold on the spatial scales that define the scattering (a few Debye lengths), but it does not need to hold across the entire scattering volume.

$$\langle n_{\boldsymbol{k}}(x, y; Z_0) n^*_{\boldsymbol{k}}(x', y'; Z_0)\rangle = \left\langle |\Delta n|^2 \right\rangle \delta(x - x')\delta(y - y') \tag{10}$$

Introduce the angles of integration

$$\theta_x = x/Z_0, \quad \theta_y = y/Z_0 \tag{11}$$

and obtain

$$\begin{aligned}\rho_{ij} = & \frac{C^2}{Z_0^2} \iint \left\langle |\Delta n(\theta_x, \theta_y)|^2 \right\rangle G^2_{\mathrm{tx}}(\theta_x, \theta_y) G_{\mathrm{rx},i}(\theta_x, \theta_y) G_{\mathrm{rx},j}(\theta_x, \theta_y) \\ & e^{2\pi i(A_{ij}\theta_x + B_{ij}\theta_y)}\, d\theta_x\, d\theta_y \end{aligned} \tag{12}$$

Expressions are obtained for the spatial auto-correlations in each of the antennas in a similar way:

$$\langle |f_{\text{rx},i}|^2 \rangle = \frac{C^2}{Z_0^2} \iint \left\langle |\Delta n(\theta_x, \theta_y)|^2 \right\rangle G_{\text{tx}}^2(\theta_x, \theta_x) G_{\text{rx},i}^2(\theta_x, \theta_x) \, d\theta_x \, d\theta_y, \quad (13)$$

and for the transmitting antenna,

$$\langle |f_{\text{tx}}|^2 \rangle = \frac{C^2}{Z_0^2} \iint \left\langle |\Delta n(\theta_x, \theta_y)|^2 \right\rangle G_{\text{tx}}^4(\theta_x, \theta_x) \, d\theta_x \, d\theta_y. \quad (14)$$

The observable normalized complex cross-correlation, also called the *complex coherence function*, is obtained through normalising (12) by the geometrical mean of factors like (13) and (14),

$$\gamma_{ij} = \frac{\rho_{ij}}{\sqrt{\langle |f_{\text{rx},i}|^2 \rangle \langle |f_{\text{rx},j}|^2 \rangle}}, \quad (15)$$

substituting f_{tx} for one of $f_{\text{rx},i}$ as appropriate if the transmitting antenna is also used as one of the receiving modules.

3.3 Wide antenna beams

Assume for mathematical convenience that a discrete scattering structure has a Gaussian shape centered at $\boldsymbol{\theta}_o = (\theta_{xo}, \theta_{yo})$ with unequal widths in the x- and y-directions, σ_x and σ_y respectively:

$$\left\langle |\Delta n(\theta_x, \theta_y)|^2 \right\rangle = \exp\left[-\frac{(\theta_x - \theta_{xo})^2}{2\sigma_x^2} - \frac{(\theta_y - \theta_{yo})^2}{2\sigma_y^2} \right] \quad (16)$$

and that the antenna beams are much wider than the scatterer. In this limit, the antenna gains in expressions (12) and (13) can be replaced with the gains in the direction of the centre of the scatterer, and taken outside the integrals, where they will eventually disappear in the normalisation at (15). When equation (16) is inserted in (12), the integral over θ_x becomes

$$\begin{aligned} \rho_{ij} &\propto \int \exp\left[-\frac{(\theta_x - \theta_{xo})^2}{2\sigma_x^2} + 2\pi i A_{ij} \theta_x \right] d\theta_x \\ &= \sqrt{2\pi} \sigma_x e^{2\pi i A_{ij} \theta_{xo}} \exp\left[-\frac{1}{2} (2\pi)^2 A_{ij}^2 \sigma_x^2 \right] \end{aligned} \quad (17)$$

with a similar result for the integration over θ_y. The resulting complex cross-correlation becomes

$$\rho_{ij} = 2\pi\sigma_x\sigma_y \frac{C^2}{Z_0^2} G_{\rm tx}^2(\theta_{xo},\theta_{yo}) G_{{\rm rx},i}(\theta_{xo},\theta_{yo}) G_{{\rm rx},j}(\theta_{xo},\theta_{yo}) \cdot e^{2\pi i \boldsymbol{D}_{ij}\cdot\boldsymbol{\theta}_o} \exp\left[-\frac{(2\pi)^2}{2}(A_{ij}^2\sigma_x^2 + B_{ij}^2\sigma_y^2)\right], \tag{18}$$

the spatial auto-correlation in each antenna is

$$\langle |f_{{\rm rx},i}|^2\rangle = G_{\rm tx}^2(\theta_{xo},\theta_{yo}) G_{{\rm rx},i}^2(\theta_{xo},\theta_{yo}) \frac{C^2}{Z_0^2} 2\pi\sigma_x\sigma_y, \tag{19}$$

which gives the following simple expression for the complex coherence:

$$\gamma_{ij} = e^{2\pi i \boldsymbol{D}_{ij}\cdot\boldsymbol{\theta}_o} \exp\left[-\frac{(2\pi)^2}{2}(A_{ij}^2\sigma_x^2 + B_{ij}^2\sigma_y^2)\right]. \tag{20}$$

This result is the two-dimensional equivalent to equation (8) in Farley et al. (1981). From this expression, the interferometry *fringe size* is defined as $1/D_{ij}$, the angular distance which corresponds to a phase shift of 2π for a scatterer observed at this interferometric baseline.

A similar result will arise from any sort of localised scattering structure, as long as it is much narrower than the antenna beams.

3.4 Gaussian antennas

Next, assume Gaussian patterns for the two antennas

$$G_{{\rm rx},i}(\theta_x,\theta_y) = \exp\left[-\frac{\theta_x^2+\theta_y^2}{2\sigma_{{\rm rx},i}^2}\right] \tag{21}$$

and the Gaussian expression (16) will again be used for the scatterer. Inserting these expressions in (12) and reorganising terms produces

$$\begin{aligned}\rho_{ij} = {} & \frac{C^2}{Z_0^2} e^{-\theta_{xo}^2/2\sigma_x^2} e^{-\theta_{yo}^2/2\sigma_y^2} \\ & \times \int \exp\left[-\frac{1}{2}\frac{\theta_x^2}{\Sigma_{ij,x}^2} + 2\left(\frac{\theta_{xo}}{2\sigma_x^2} + i\pi A_{ij}\right)\theta_x\right] d\theta_x \\ & \times \int \exp\left[-\frac{1}{2}\frac{\theta_y^2}{\Sigma_{ij,y}^2} + 2\left(\frac{\theta_{yo}}{2\sigma_y^2} + i\pi B_{ij}\right)\theta_y\right] d\theta_y\end{aligned}$$

where

$$\frac{1}{\Sigma_{ij}^2} = \frac{2}{\sigma_{\rm tx}^2} + \frac{1}{\sigma_{{\rm rx},i}^2} + \frac{1}{\sigma_{{\rm rx},j}^2}, \qquad \frac{1}{\Sigma_{ij,x}^2} = \frac{1}{\Sigma_{ij}^2} + \frac{1}{\sigma_x^2} \tag{22}$$

and $\Sigma^2_{ij,y}$ defined equivalently.

Here, the integrals over $\theta_{x,y}$ have been put in a form found in tables, and the resulting complex cross-correlation is

$$\begin{aligned}\rho_{ij} = \frac{C^2}{Z_0^2} 2\pi \Sigma_{ij,x}\Sigma_{ij,y} \exp\left[-\frac{1}{2}\frac{\Sigma^2_{ij,x}}{\Sigma^2_{ij}}\frac{\theta^2_{xo}}{\sigma^2_x} - \frac{1}{2}\frac{\Sigma^2_{ij,y}}{\Sigma^2_{ij}}\frac{\theta^2_{yo}}{\sigma^2_y}\right] \\ \times \exp\left[-\frac{1}{2}(2\pi)^2 A^2_{ij}\Sigma^2_{ij,x}\right] \exp\left[-\frac{1}{2}(2\pi)^2 B^2_{ij}\Sigma^2_{ij,y}\right] \\ \times \exp\left[2\pi i\left\{A_{ij}\theta_{xo}\frac{\Sigma^2_{ij,x}}{\sigma^2_x} + B_{ij}\theta_{yo}\frac{\Sigma^2_{ij,y}}{\sigma^2_y}\right\}\right]\end{aligned} \tag{23}$$

Similarly, the spatial auto-correlation in each antenna becomes

$$\langle|f_{\mathrm{rx},i}|^2\rangle = \frac{C^2}{Z_0^2} 2\pi\Sigma_{i,x}\Sigma_{i,y} \exp\left[-\frac{1}{2}\frac{\Sigma^2_{i,x}}{\Sigma^2_i}\frac{\theta^2_{x0}}{\sigma^2_x}\right] \exp\left[-\frac{1}{2}\frac{\Sigma^2_{i,y}}{\Sigma^2_i}\frac{\theta^2_{y0}}{\sigma^2_y}\right] \tag{24}$$

where

$$\frac{1}{\Sigma^2_i} = \frac{2}{\sigma^2_{\mathrm{tx}}} + \frac{2}{\sigma^2_{\mathrm{rx},i}}, \qquad \frac{1}{\Sigma^2_{i,x}} = \frac{1}{\Sigma^2_i} + \frac{1}{\sigma^2_x} \tag{25}$$

with similar expressions for $\Sigma_{i,y}$ and for antenna j.

The expression for complex coherence then becomes

$$\begin{aligned}\gamma_{ij} = \frac{\Sigma_{ij,x}\Sigma_{ij,y}}{(\Sigma_{i,x}\Sigma_{i,y}\Sigma_{j,x}\Sigma_{j,y})^{1/2}} \\ \times \frac{\exp\left[-\frac{1}{2}\frac{\Sigma^2_{ij,x}}{\Sigma^2_{ij}}\frac{\theta^2_{xo}}{\sigma^2_x} - \frac{1}{2}\frac{\Sigma^2_{ij,y}}{\Sigma^2_{ij}}\frac{\theta^2_{yo}}{\sigma^2_y}\right]}{\exp\left[-\frac{1}{2}\left(\frac{\Sigma^2_{i,x}}{\Sigma^2_i} + \frac{\Sigma^2_{j,x}}{\Sigma^2_j}\right)\frac{\theta_{xo}}{\sigma^2_x} - \frac{1}{2}\left(\frac{\Sigma^2_{i,y}}{\Sigma^2_i} + \frac{\Sigma^2_{j,y}}{\Sigma^2_j}\right)\frac{\theta_{yo}}{\sigma^2_y}\right]^{1/2}} \\ \times \exp\left[-\frac{1}{2}(2\pi)^2 A^2_{ij}\Sigma^2_{ij,x}\right] \exp\left[-\frac{1}{2}(2\pi)^2 B^2_{ij}\Sigma^2_{ij,y}\right] \\ \times \exp\left[2\pi i\left\{A_{ij}\theta_{xo}\frac{\Sigma^2_{ij,x}}{\sigma^2_x} + B_{ij}\theta_{yo}\frac{\Sigma^2_{ij,y}}{\sigma^2_y}\right\}\right]\end{aligned} \tag{26}$$

but this expression simplifies enormously when the receive antennas are equal, or when the size of the scatterer is much smaller than the antenna beams, as we will se below.

The normalisation in (15) removes the absolute scattering levels and most of the effect due to tapering antenna patterns (the exponentials over $-\theta^2_{xo,yo}$) from the expressions. Any remaining difference from the simple expression (20) in the resulting coherence for a given pattern and structure size is due to the exponents in the remaining two exponentials in (26) being multiplied by $\Sigma^2_{ij,x}/\sigma^2_x$ or $\Sigma^2_{ij,y}/\sigma^2_y$.

3.5 Equal Gaussian receive antennas

When the receiver antennas are equal, $\Sigma_i = \Sigma_j = \Sigma_{ij}$, $\Sigma_{i,x} = \Sigma_{j,x} = \Sigma_{ij,x}$, etc., the normalization factors in the first two lines of eq. (26) cancel exactly, and the expression for the complex coherence is greatly simplified,

$$\gamma_{ij} = \exp\left[-\frac{1}{2}(2\pi)^2(A_{ij}^2\Sigma_{ij,x}^2 + B_{ij}^2\Sigma_{ij,y}^2)\right]\exp\left[2\pi i\left\{A_{ij}\theta_{xo}\frac{\Sigma_{ij,x}^2}{\sigma_x^2} + B_{ij}\theta_{yo}\frac{\Sigma_{ij,y}^2}{\sigma_y^2}\right\}\right]. \tag{27}$$

3.6 Narrow scatterers

If the scattering structure is assumed to be much smaller than the beams of the receive antennas, which means $\Sigma_{ij} \gg \sigma_x, \sigma_y$, then $\Sigma_{ij,x} \approx \sigma_x$ and $\Sigma_{ij,y} \approx \sigma_y$. In this case, the complex cross-correlation parts (the last two lines) of expression (26) can be simplified somewhat.

For equal receive antennas, eq. (27) simplifies further to

$$\gamma_{ij} = e^{2\pi i \boldsymbol{D}_{ij}\cdot\boldsymbol{\theta}_o}\exp\left[-\frac{(2\pi)^2}{2}(A_{ij}^2\sigma_x^2 + B_{ij}^2\sigma_y^2)\right], \tag{28}$$

which is the same result as arrived at earlier (20).

4 Discussion

The antenna positions have been given as in the aperture plane, a plane perpendicular to the beam direction of the antennas. In practice, this will almost always be a projected position, which will change with the direction of the transmitted beam. In other words, the baselines are dependent on the pointing direction.

Expression (28), or the slightly more accurate expression (27), are the two-dimentional equivalents to eq. (8) by Farley et al. (1981), as mentioned above. The coherence is determined by the structure's size, projected along the baseline, with a small modification due to the finite antenna size. The phase is determined by the position of the scatterer, also this one projected along the direction of the baseline, and with a small modification due to the finite antenna size.

In section 3.5, it was shown that the assumption of equal receiving antennas causes exact cancellation of normalization factors (shown only for Gaussian patterns, but more generally valid), but even for moderately unequal patterns the effect of these normalization factors is not large.

Assuming a scattering structure which is narrow with respect to the antenna beams causes significant simplifications in the expressions. For the phenomena to be studied with the radar interferometer, e.g. aurora, they can be quite extended, sometimes covering the visible sky. The possible source of localized scattering, however, is limited by the beam of the transmitting antenna, which is likely to be much narrower than the beams of the receiving antennas. For practical purposes, this means that scattering is likely to occur only in a limited angular region.

5 Summary

We have developed the basic expressions for the observable complex cross-correlation and complex coherence in a two-element radar interferometer with an external transmit antenna. The expressions are two-dimensional generalizations of the corresponding expressions used in earlier one-dimensional setups. In the earlier results, one of the receive antennas was also used to transmit, while in this setup a separate transmit antenna is considered. This also causes some modifications in the structure of the expressions.

The development is straightforward but involved, and the resulting expressions are cumbersome, but significant simplifications are possible for most cases of interest. We have looked at these simplifications and how to interpret the resulting expressions.

In the next reports we will look at the techniques available to recreate an *image* of the scattering structures from a set of measured coherences and phases. We will also investigate how the performance of the image inversion techniques bear upon the size and form of each receiving antenna or module and their number and geometric configuration. Other important issues that will be analysed are: relative phase accuracy requirements and calibration procedures, among others.

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