Radar Interferometer Phase Calibration using the Visibility function of Incoherent Scattering

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Abstract
The visibility function of incoherent scattering signals from the ionosphere is calculated. Under conditions that are usually fulfilled in actual experiments it is shown that the visibility function is a real function, therefore the visibility phase is constant and equal to zero. As was suggested by Grydeland et al. (2007), this affords a practical procedure to make global relative calibration of the baseline phases of synthetic aperture radar interferometer used for ionospheric imaging. An example is given of actual measurements made with the two antenna system of the EISCAT Svalbard Radar.

From La Hoz et al. (2002) and Grydeland and La Hoz (2006) the visibility function is:

$$V(A) = \frac{\langle f_1 f_2^* \rangle}{\sqrt{\langle |f_1|^2 \rangle \langle |f_2|^2 \rangle}}$$

where A is the baseline length in wavelength units;

$$f_1 = \mathrm{const} \int \int n(x,y;k) e^{-2\pi \mathrm{i}\left(x^2+y^2\right)/Z_o} g_1^2(x,y) dx dy$$

$$f_2 = \mathrm{const} \int \int n(x,y;k) e^{-2\pi \mathrm{i}\left(x^2+y^2+\mathrm{Ax}\right)/Z_o} g_1(x,y) g_2(x,y) dx dy$$

$$\langle f_1 f_2^* \rangle = |\mathrm{const}|^2 \int \int \left\langle |\Delta \mathrm{n}(x,y;k)|^2 \right\rangle e^{2\pi \mathrm{iAx}} g_1^3(x,y) g_2(x,y) dx dy$$

where the g's are amplitude antenna patterns. We specialize to the ESR case: Parabolic dishes, assume they are identical for simplicity:

$$g_1 = g_2 = g$$

$$g^2\left(\theta_x, \theta_y\right) = G\left(\theta_x, \theta_y\right) = e^{-\frac{\theta_x^2+\theta_y^2}{2\sigma^2}}$$

where G is the antenna (power) pattern. Using the condition that neighbouring points of the electron density are uncorrelated results in:

$$\left\langle |f_1|^2 \right\rangle = \left\langle |f_2|^2 \right\rangle = \mathrm{const}^2 \int\int \left\langle |\Delta \mathrm{n}(x,y;k)|^2 \right\rangle G^2(x,y)dxdy$$

Assuming that the electron density is homogeneous within the scattering volume, which is generally the case:

$$\left\langle |\Delta n(x,y;k)|^2 \right\rangle = \left\langle |\Delta \mathrm{n}|^2 \right\rangle$$

$$\sqrt{\langle |f_1|^2 \rangle \left\langle |f_2|^2 \right\rangle} = \mathrm{const}^2 \left\langle |\Delta \mathrm{n}|^2 \right\rangle \int\int G^2(x,y)dxdy$$

$$\langle f_1 f_2^* \rangle = |\mathrm{const}|^2 \left\langle |\Delta \mathrm{n}|^2 \right\rangle \int\int e^{2\pi \mathrm{iAx}} G^2(x,y)dxdy$$

$$V(A) = \frac{\langle f_1 f_2^* \rangle}{\sqrt{\left\langle |f_1|^2 \right\rangle \left\langle |f_2|^2 \right\rangle}} = \frac{\int\int e^{2\pi \mathrm{iAx}} G^2(x,y)dxdy}{\int\int G^2(x,y)dxdy}$$

$$= \frac{\int_{-\infty}^{\infty} e^{-\frac{x^2}{\sigma^2}} e^{2\pi \mathrm{iAx}} dx}{\int_{-\infty}^{\infty} e^{-\frac{x^2}{\sigma^2}}\, dx} = \frac{\mathrm{FT}\left\{ e^{-\frac{x^2}{\sigma^2}} \right\}}{\sqrt{\pi}\sigma}$$

$$= \frac{\sigma e^{-A^2\pi^2 s^2}\sqrt{\pi}}{\sqrt{\pi}\sigma} = e^{-A^2\pi^2\sigma^2}$$

$$\mathbf{FourierTransform}\left[\mathbf{Exp}\left[-\frac{x^2}{s^2}\right], x, A, \mathbf{FourierParameters} \to \{0, 2\pi\}\right]$$

$$\frac{e^{-A^2\pi^2 s^2}\sqrt{\pi}}{\sqrt{\frac{1}{s^2}}}$$

$$\int_{-\infty}^{\infty} e^{-x^2/s^2}\, dx$$

$$\mathrm{If}\left[\mathrm{Re}\left[s^2\right] > 0, \sqrt{\pi}\sqrt{s^2}, \mathrm{Integrate}\left[e^{-\frac{x^2}{s^2}}, \{x, -\infty, \infty\}, \mathrm{Assumptions} \to \mathrm{Re}\left[s^2\right] \le 0\right]\right]$$

$$\mathbf{Simplify}\left[\frac{s e^{-A^2\pi^2 s^2}\sqrt{\pi}}{s\sqrt{\pi}}\right]$$

$$e^{-A^2\pi^2 s^2}$$

The main result is the visibility function:

$$V(A) = e^{-A^2\pi^2\sigma^2}$$

The Visibility Phase is always zero provided the electron density is uniform within the scattering volume and the antenna pattern is symmetric across the baseline. More generally, when the product of the electron density and the antenna pattern is symmetric within the scattering volume across the baseline.

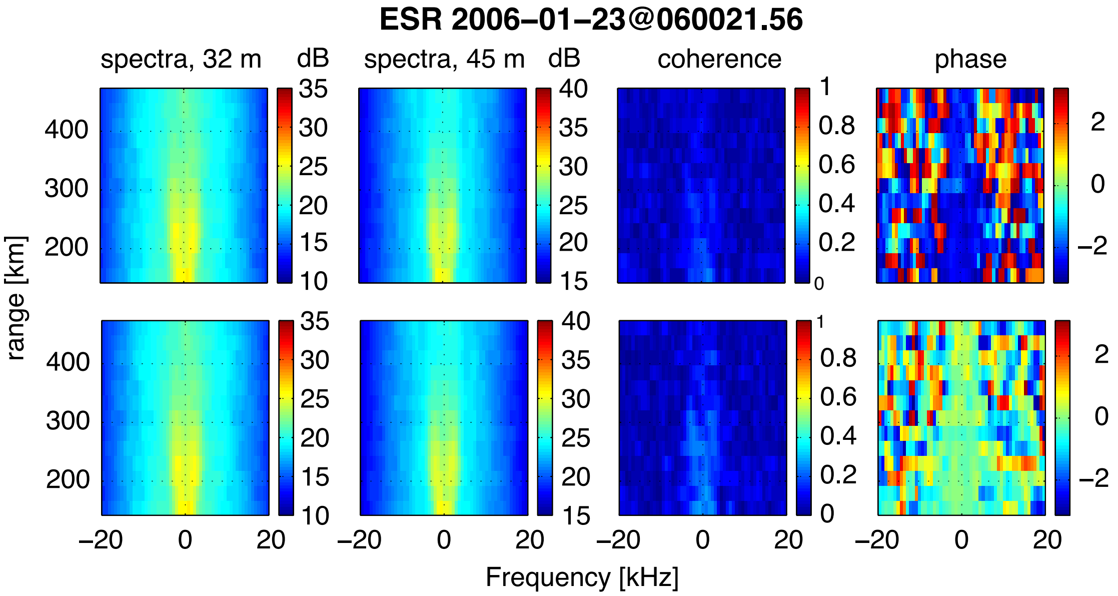

Figure 1: Spectra and complex coherence measured by the ESR. The phase in the lower panel has been calibrated by subtracting the phase measured in the first panel.

The symmetry condition assures that the visibility function is a real function.

According to www.EISCAT.com the full width at half power of the 32 m antenna is 0.6 degrees:

$\boldsymbol{\sigma_{1/2} = 0.3 \times \frac{\pi}{180.}} = 0.00523599$
$\boldsymbol{\sigma = \frac{\sigma_{1/2}}{\sqrt{2.\mathrm{Log}[2.]}}} = 0.00444704$

$\mathbf{visibility[bl\_, bw\_]{:=}}\boldsymbol{N}\,\mathbf{\left[Exp\left[-\pi^2 bl^2 bw^2\right]\right]}$
ESR32bw = 0.00444704; (* Gaussian beamwidth σ *)
ESRbl = 128./0.599585 = 213.481 (* Baseline in wavelengths *)
visibility[ESRbl, ESR32bw] = 0.000137032

The constant IS Visibility of the two ESR antennas assuming both identical with radius 32 m is:

$$V = 1.4 \times 10^{-4}$$

The actual visibility with the combination of the 32 and 45 m antennas is a bit larger.

Grydeland et al. (2006, 2007) discovered that the coherence function of incoherent scattering signals from the ionosphere exhibits very low amplitude and very well ordered phase — meaning constant phase — within the bandwidth of the IS signal. This is shown in figure 1.

He suggested that this finding could be used as a means to calibrate a radar interferometer, which is justified by the proof shown in this report. The upper panel shows from left to right two frames with spectrograms of IS signals measured by the 32 and 45 metre antennas of the EISCAT Svalbard Radar, respectively, while the third and fourth frames show the amplitude and phase of the coherence function of the two signals as a function of Doppler frequency. It is clear in the last frame that the phases are constant within the bandwidth of the IS signals, as this region is uniformly dark blue. The lower panel shows the result after calibration, namely after the constant phase of the upper panel has been subtracted.

References

Grydeland, T., and C. La Hoz, WP5: Fundamentals of radar interferometry I: One baseline. EISCAT_3D Stage 2 report, April, 2006.

Grydeland, T., C. La Hoz and V. Belyey, Phase calibration of an aperture synthesis imaging array by means of incoherent scattering, 11th International Workshop on Technical and Scientific aspects on MST RADAR, Gadanki/Tirupati, India, December 2006.

Grydeland, T., C. La Hoz and V. Belyey, Phase calibration of an aperture synthesis imaging array by means of incoherent scattering, Abstract, 13th EISCAT International Workshop August 6-10, Åland, Finland, 2007.

La Hoz, C., Hagfors, T., and Grydeland, T.: A search for small scale structure within the antenna beam using the EISCAT Svalbard Radar dual antennas in an interferometer mode, internal report, University of Troms001C, Department of Physics,, 2002.